

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019

TSC2571 – SCIENTIFIC COMPUTING (TC01 / TT01)

25 OCTOBER 2018
9.00 a.m - 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 5 pages (inclusive of the cover page).
2. Answer **ALL** questions. The distribution of marks for each question is given.
3. The **Appendix** section begins from **page 4 onwards**.
4. Please write all your answers in the Answer Booklet provided.
5. Please write the answers for each question on a new page of your Answer Booklet.

QUESTION 1: ROOTS OF EQUATIONS**[Total Marks: 7]**

Given an equation as follows,

$$f(x) = \sin x + e^x$$

Using an initial value of $a = 0$, apply Newton's method to find a root for the function with a tolerance of 1×10^{-5} . Write your answers in 5 decimal places. [7 marks]

QUESTION 2: NUMERICAL INTEGRATION**[Total Marks: 8]**

Given an integral as follows,

$$\int_0^{0.5} \sqrt{2x+1} \, dx$$

- (a) Use the Simpson's Rule with 4 equal parts ($n = 4$) to evaluate this integral. Write your answers in 6 decimal places. [7 marks]
- (b) Given that the actual value is 0.609476, calculate the relative error between the actual value and your answer in (a). [1 mark]

QUESTION 3: LEAST SQUARES PROBLEMS, INTERPOLATION AND POLYNOMIAL APPROXIMATION**[Total Marks: 10]**

Given the following data sets,

x	-4	-1.5	0	2	5	8
y	-20	-1	2	12	42	76

- (a) Find the best fit line of $y = a + bx$ using the method of least squares. Write your answers in 3 decimal places. [9 marks]
- (b) Using your answer in (a), find y for $x = 10$. Write your answer in 3 decimal places. [1 mark]

Continued...

QUESTION 4: MATRICES AND SYSTEMS OF LINEAR EQUATIONS**[Total Marks: 15]**

(a) Given the following systems of linear equations,

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Solve for x, y, z using the method of Gaussian Elimination with Back Substitution.

[7 marks]

(b) For the following systems of linear equations,

$$3u + v = -2$$

$$u + 8v = 1$$

- (i) Find the values of u and v using both **Jacobi Method** and **Gauss-Siedel Method with 2 iterations**. Begins your iteration with $u_0 = 0, v_0 = 0$. Round off your answers to 6 decimal places.
- (ii) Calculate the absolute error between the two methods for each u and v that you get in (i):

[6 + 2 = 8 marks]

End of Questions.

APPENDIX: USEFUL FORMULAS

ROOTS OF EQUATION

Bisection Method	Secant Method	Newton-Raphson Method
$p_n = \frac{b_n + a_n}{2}$	$p_{n+1} = p_n - \frac{f(p_n)(p_n - p_{n-1})}{f(p_n) - f(p_{n-1})}$	$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}$

NUMERICAL INTEGRATION

Trapezoidal Rule
$A_T = \int_a^b f(x) dx$ $\approx \frac{h}{2} \{ f(x_1) + f(x_{n+1}) + 2[f(x_2) + f(x_3) + \dots + f(x_n)] \}$ <p>where $x_n = a + (n-1)h$, $n = 1, 2, \dots$</p>

Simpson's Rule
$A_T = \int_a^b f(x) dx$ $\approx \frac{h}{3} \left\{ f(x_1) + f(x_{2n+1}) + 4[f(x_2) + f(x_4) + \dots + f(x_{2n})] \right. \\ \left. + 2[f(x_3) + f(x_5) + \dots + f(x_{2n-1})] \right\}$ <p>where $x_n = a + (n-1)h$, $n = 1, 2, \dots$</p>

Romberg Algorithm
$R(0,0) = \frac{1}{2}(b-a)[f(a) + f(b)]$ $R(n,0) = \frac{1}{2}R(n-1,0) + h \sum_{k=1}^{2^{n-1}} f(a + (2k-1)h); \quad h = \frac{(b-a)}{2^n}, \quad n \geq 1$ $R(n,m) = R(n,m-1) + \frac{1}{4^m - 1} [R(n,m-1) - R(n-1,m-1)],$ <p>where $n \geq 1$, $m \geq 1$, $m \leq n$</p>

LEAST SQUARES PROBLEMS, INTERPOLATION AND POLYNOMIAL APPROXIMATION

Lagrange Coefficient

$$L_i(x) = \frac{(x-x_0)(x-x_1)\dots(x-x_{i-1})(x-x_{i+1})\dots(x-x_n)}{(x_i-x_0)(x_i-x_1)\dots(x_i-x_{i-1})(x_i-x_{i+1})\dots(x_i-x_n)}$$

where $i = 0, 1, \dots, n$

Linear Least Squares

$$y = a + bx$$

$$a = \frac{\sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i - \sum_{i=1}^n x_i y_i \sum_{i=1}^n x_i}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2} \quad b = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2}$$

$$y = ae^{bx}$$

$$A = \frac{\sum_{i=1}^n x_i^2 \sum_{i=1}^n Y_i - \sum_{i=1}^n x_i Y_i \sum_{i=1}^n x_i}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2} \quad b = \frac{n \sum_{i=1}^n x_i Y_i - \sum_{i=1}^n x_i \sum_{i=1}^n Y_i}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2}$$

$$y = ax^b$$

$$A = \frac{\sum_{i=1}^n X_i^2 \sum_{i=1}^n Y_i - \sum_{i=1}^n X_i Y_i \sum_{i=1}^n X_i}{n \sum_{i=1}^n X_i^2 - \left(\sum_{i=1}^n X_i \right)^2} \quad b = \frac{n \sum_{i=1}^n X_i Y_i - \sum_{i=1}^n X_i \sum_{i=1}^n Y_i}{n \sum_{i=1}^n X_i^2 - \left(\sum_{i=1}^n X_i \right)^2}$$

End of Page.